PHYSICAL REVIEW E VOLUME 48, NUMBER 4 OCTOBER 1993

Depolarization of light by an imperfect polarizer

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(Received 1 March 1993)

The intensity of originally unpolarized light passing through three linear polarizers from which the first two were identical and crossed was shown to be fully independent of the azimuthal angle of the third (testing) polarizer. This means that the second polarizer acts as a depolarizer of light polarized by the first one. We interpret the phenomenon phenomenologically on the basis of Mueller calculus in which polarized light is described by Stokes vectors.

PACS number(s): 42.25.Ja

A light beam having passed through an ideal linear polarizer is assumed to be polarized fully in one plane so that no light can pass through the other ideal polarizer (analyzer) oriented orthogonally to the first polarizer. As for real polarizers some light always passes through such a polarizer pair. The reason is seen in the imperfection of real polarizers, which is defined by the so-called polarizer defect

$$G_d = \frac{k_2}{k_1+k_2} , \tag{1}$$

where the coefficients k_1 and k_2 characterize major and minor principal transparencies (see, e.g., [1]). The parameter k_1 represents the transmittance of light if it is polarized in the direction of transmission axis and k_2 represents the transmittance of light polarized in the orthogonal direction.

Total transmittance of unpolarized light through an imperfect polarizer is taken to be equal to [1]

$$\tau = \tfrac{1}{2}(k_1+k_2) . \tag{2}$$

The transmittance of light through a pair of polarizers (A,B) is then described with the help of the generalized Malus law. The minimum transmittance (in the case of exactly crossed polarizers) equals

$$\tau_{AB\min} = \tfrac{1}{2}(k_{A1}k_{B2}+k_{A2}k_{B1}) . \tag{3}$$

This value is smaller the less imperfect are the polarizers used (i.e., the smaller are the coefficients k_2). That may be regarded as a main reason why sufficient attention has not been devoted to the actual polarization degree of light passing through the pair of crossed polarizers. One has usually assumed that the outgoing light is polarized in the direction of the transmittance axis of the second (last) polarizer as supported by the description of polarized light with the help of Jones vectors (see, e.g., [1]). In the following we will try to show that the polarization degree is a function of the coefficients k of all polarizers involved, allowing the transmitted light to be fully unpolarized in special cases.

A general quasimonochromatic light beam may consist of polarized and unpolarized parts. For the description of such properties Mueller calculus is standardly used. The light beam is characterized by elements of Stokes vectors $S \equiv \{S_0, S_1, S_2, S_3\}$ while the effect of a polarizer is described with the help of a Mueller (4×4) matrix; S_0 is the total intensity of the light beam, S_1 is the difference between the intensities of two components polarized in the basic orthogonal axis, S_2 is the same but the axes are shifted by 45°, and S_3 represents the circularly polarized component [2]. The Mueller matrix describing a linear polarizer characterized by the coefficients k_1 and k_2 may be written as

$$M = \frac{1}{2}\begin{vmatrix} k_1+k_2 & k_1-k_2 & 0 & 0 \\ k_1-k_2 & k_1+k_2 & 0 & 0 \\ 0 & 0 & 2\sqrt{k_1k_2} & 0 \\ 0 & 0 & 0 & 2\sqrt{k_1k_2} \end{vmatrix} . \tag{4}$$

If a fully unpolarized beam passes through such a polarizer the properties of the outgoing beam may be characterized by the Stokes vector

$$S_{\text{out}} = \tfrac{1}{2}\{k_1+k_2, k_1-k_2, 0, 0\} . \tag{5}$$

The polarization degree is then equal to (see, e.g., [2])

$$P = \frac{S_1}{S_0} = \frac{k_1-k_2}{k_1+k_2} , \tag{6}$$

which means that the amount of unpolarized light in the outgoing beam equals k_2 (related to the unit intensity of the incoming beam). The portion of unpolarized light in the outgoing beam is then

$$r = \frac{k_2}{\frac{1}{2}(k_1+k_2)} = 2G_d , \tag{7}$$

which shows that the polarizer imperfectness [defined by Eq. (1)] may be related directly to this portion.

The polarization degree of the light beam leaving the pair of exactly crossed polarizers (i.e., in the case of minimum transmittance) is then equal according to Mueller calculus to

$$P = \frac{|k_{A1}k_{B2}-k_{A2}k_{B1}|}{k_{A1}k_{B2}+k_{A2}k_{B1}} . \tag{8}$$

One can see immediately that the polarization degree is zero if the pair consists of two identical polarizers. Of course, it equals zero always if

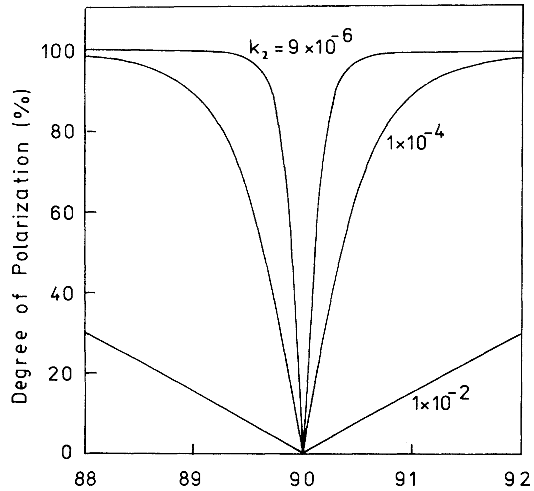

FIG. 1. Angle dependence of degree of polarization in the case of a pair of identical polarizers calculated for $k_1=0.81$ and for different values of k_2.

$$\frac{k_{A2}}{k_{A1}}=\frac{k_{B2}}{k_{B1}} . \tag{9}$$

In such a case the analyzer (i.e., the second polarizer) diminishes the polarization degree P given by Eq. (6) to zero and acts as a depolarizer. As far as we know such a possibility has not yet been mentioned in the literature. The polarization degree of light outgoing from a pair of identical polarizers depends, of course, on the angle between their axes as shown for different values of k_2 in Fig. 1.

We tried to verify the mentioned depolarization effect of the analyzer experimentally and to measure the light intensity of the beam passing through a system of three polarizers when the first two were exactly crossed. The dependence of the transmittance on the angle of the third polarizer is shown in Fig. 2. The multilayer dielectric polarizers used were chosen to be of lower quality to ensure easier verification of the predicted phenomenon:

$$\begin{aligned} &k_{A1}=0.893\pm0.013, \quad k_{A2}=0.0146\pm0.0012 ,\\ &k_{B1}=0.903\pm0.013, \quad k_{B2}=0.0135\pm0.0012 ,\\ &k_{C1}=0.767\pm0.011, \quad k_{C2}=0.0331\pm0.0018 . \end{aligned} \tag{10}$$

The light beam leaving the given pair of crossed polarizers may be regarded as depolarized as its modulation can be neglected in relation to the measured intensity, being of the order of measurement errors. The possibility of the light being elliptically polarized was excluded by using a retarder.

It follows from the preceding that the analyzer may exhibit an unexpected function, i.e., it can fully depolarize a partially polarized light beam [3].

Let us characterize such a beam by the Stokes vector

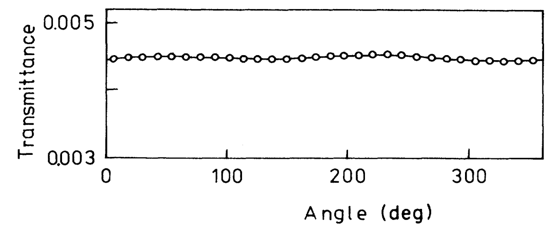

FIG. 2. Measured dependence of the transmittance of the system of three linear polarizers on the azimuth angle of the last polarizer when the first two polarizers are in the crossed configuration.

$$S_{\text{in}}=\{1,P_{\text{in}},0,0\} , \tag{11}$$

where the polarization degree $P_{\text{in}}<1$. If the polarization plane of light and the transmission axis of a polarizer are at the angle $\alpha=90°$ then the effect of the polarizer may be represented by the Mueller matrix

$$M'=\frac{1}{2}\begin{vmatrix} k_1+k_2 & -(k_1-k_2) & 0 & 0\\ -(k_1-k_2) & k_1+k_2 & 0 & 0\\ 0 & 0 & 2\sqrt{k_1k_2} & 0\\ 0 & 0 & 0 & 2\sqrt{k_1k_2} \end{vmatrix} . \tag{12}$$

The transmitted light may be then described by the Stokes vector

$$S_{\text{out}}=\tfrac{1}{2}\{k_1+k_2-P_{\text{in}}(k_1-k_2),P_{\text{in}}(k_1+k_2) -(k_1-k_2),0,0\} . \tag{13}$$

Its polarization degree P_{out} equals zero if the parameters k_1 and k_2 characterizing the polarizer fulfill the condition

$$\frac{k_1-k_2}{k_1+k_2}=P_{\text{in}} . \tag{14}$$

Independently of Eq. (14) being satisfied P_{out} is always less than P_{in}. In the opposite case when $\alpha=0°$ (the maximum transmittance being reached) P_{out} is greater than P_{in}. It holds

$$\frac{P_{\text{out}}}{P_{\text{in}}}=\frac{|k_1+k_2\mp(k_1-k_2)/P_{\text{in}}|}{k_1+k_2\mp(k_1-k_2)P_{\text{in}}} , \tag{15}$$

where the upper (lower) sign corresponds to $\alpha=90°$ ($\alpha=0°$).

Even if there is a general trend to make use of very perfect polarizers the so-called imperfect ones may find also some useful applications as effective depolarizers, which may be related to some unexpected physical (optical) properties of real polarizers.

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